求模拟滤波器传递函数的极点

知识点： 求极点 $s_k=\varepsilon^{-\frac{1}{N}}\Omega_c e^{\frac{j(2k+N-1)\pi}{2N}}\qquad k=0,1,\ldots 2N-1$

模拟滤波器传递函数的极点是在 s 左半平面的极点：k=1,2,...N，令ε=1，Ωc=1

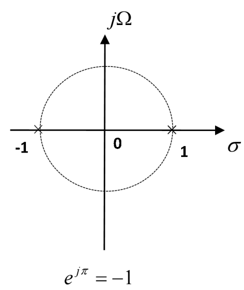
（1）N=1

$$s_k=e^{\frac{j(2k+N-1)\pi}{2N}}=e^{\frac{j(2k)\pi}{2}}=e^{jk\pi}\qquad k=0,1$$

在 s 左半平面的极点为：

$$s_1=e^{j\pi}=-1$$

一阶：N = 1



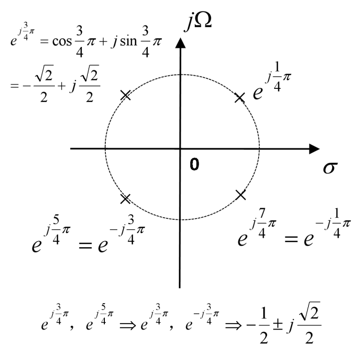
（2）N=2

$$s_k=e^{\frac{j(2k+N-1)\pi}{2N}}=e^{\frac{j(2k+1)\pi}{4}}\qquad k=0,1,2,3$$

在 s 左半平面的极点为：

$$s_1=e^{\frac{j3\pi}{4}},s_2=s_1^{*}=e^{\frac{j5\pi}{4}}=e^{-\frac{j3\pi}{4}}$$

二阶：N = 2



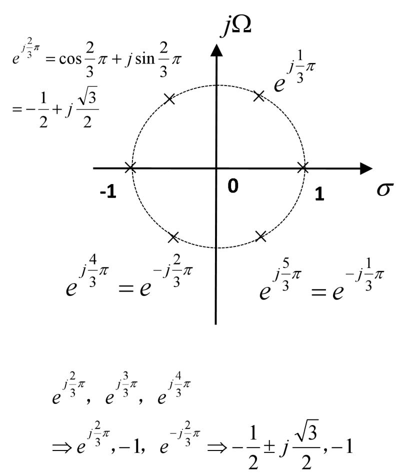
（3）N=3

$$s_k=e^{\frac{j(2k+N-1)\pi}{2N}}=e^{\frac{j(2k+2)\pi}{6}}=e^{\frac{j(k+1)\pi}{3}}\qquad k=0,1,\ldots 5$$

在 s 左半平面的极点为：

$$s_1=e^{\frac{j2\pi}{3}},s_2=e^{j\pi}=-1,s_3=s_1^{*}=e^{\frac{j4\pi}{3}}=e^{-\frac{j2\pi}{3}}$$

三阶：N = 3



(4) N=4

$$s_k = e^{\frac{j(2k+N-1)\pi}{2N}} = e^{\frac{j(2k+3)\pi}{8}} \qquad k = 0,1,\ldots 7$$

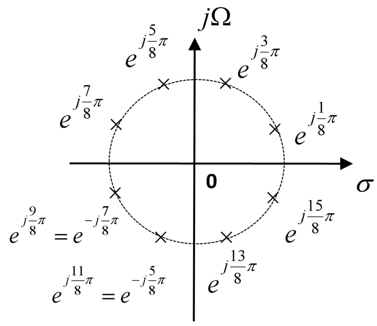
在 s 左半平面的极点为：

$$s_1 = e^{\frac{j5\pi}{8}}, s_2 = e^{\frac{j7\pi}{8}},$$

$$s_3 = s_2^{\ *} = e^{\frac{j9\pi}{8}} = e^{-\frac{j7\pi}{8}},$$

$$s_4 = s_1^{\ *} = e^{\frac{j11\pi}{8}} = e^{-\frac{j5\pi}{8}}$$

四阶：N=4



$$e^{j\frac{5}{8}\pi},\ e^{j\frac{7}{8}\pi},\ e^{j\frac{9}{8}\pi}, e^{j\frac{11}{8}\pi}$$

$$\Rightarrow e^{\pm j\frac{5}{8}\pi},\ e^{\pm j\frac{7}{8}\pi}$$

(5) N=5

$$s_k = e^{\frac{j(2k+N-1)\pi}{2N}} = e^{\frac{j(2k+4)\pi}{10}} = e^{\frac{j(k+2)\pi}{5}} \qquad k = 0,1,\ldots 9$$

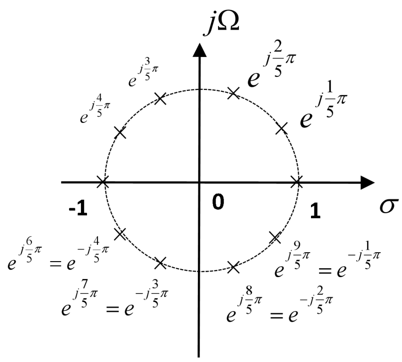
在 s 左半平面的极点为：

$$s_1 = e^{\frac{j3\pi}{5}}, s_2 = e^{\frac{j4\pi}{5}}, s_3 = e^{j\pi} = -1,$$

$$s_4 = s_2^{\ *} = e^{\frac{j6\pi}{5}} = e^{-\frac{j4\pi}{5}},$$

$$s_5 = s_1^{\ *} = e^{\frac{j7\pi}{5}} = e^{-\frac{j3\pi}{5}}$$

五阶：N=5



$$e^{j\frac{3}{5}\pi},\ e^{j\frac{4}{5}\pi},\ e^{j\pi},\ e^{j\frac{6}{5}\pi},\ e^{j\frac{7}{5}\pi}$$

$$\Rightarrow e^{\pm j\frac{3}{5}\pi}, -1,\ e^{\pm j\frac{4}{5}\pi}$$

(6) N=6

$$s_k = e^{\frac{j(2k+N-1)\pi}{2N}} = e^{\frac{j(2k+5)\pi}{12}} \qquad k = 0,1,\ldots 11$$

在 s 左半平面的极点为：

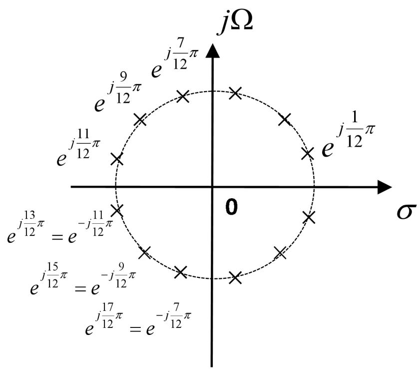
$$s_1 = e^{\frac{j7\pi}{12}}, s_2 = e^{\frac{j9\pi}{12}}, s_3 = e^{\frac{j11\pi}{12}},$$

$$s_4 = s_3^{\ *} = e^{\frac{j13\pi}{12}} = e^{-\frac{j11\pi}{12}},$$

$$s_5 = s_2^{\ *} = e^{\frac{j15\pi}{12}} = e^{-\frac{j9\pi}{12}},$$

$$s_6 = s_1^{\ *} = e^{\frac{j17\pi}{12}} = e^{-\frac{j7\pi}{12}}$$

六阶：N=6



$$e^{j\frac{7}{12}\pi}, e^{j\frac{9}{12}\pi}, e^{j\frac{11}{12}\pi}, e^{j\frac{13}{12}\pi}, e^{j\frac{15}{12}\pi}, e^{j\frac{17}{12}\pi}$$

$$\Rightarrow e^{\pm j\frac{7}{12}\pi}, e^{\pm j\frac{9}{12}\pi}, e^{\pm j\frac{11}{12}\pi}$$