连续信号的卷积

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定义：

$$y(t)=x_1(t)*x_2(t)=\int_{-\infty}^{\infty}x_1(\tau)x_2(t-\tau)d\tau$$

运算：翻转、移位、相乘、相加

性质：

交换律： $x_1(t)*x_2(t)=x_2(t)*x_1(t)$

结合律： $\big(x_1(t)*x_2(t)\big)*x_3(t)=x_1(t)*\big(x_2(t)*x_3(t)\big)$

分配律： $x_1(t)*\big(x_2(t)+x_3(t)\big)=x_1(t)*x_2(t)+x_1(t)*x_3(t)$

微积分性质：两个信号卷积的导数或积分，就等于其中任一信号的导数或积分与另一信号的卷积。

$$x^{(1)}(t)=\frac{d}{dt}x(t),\quad x^{(-1)}(t)=\int_{-\infty}^{t}x(\tau)d\tau$$

若$y(t)=x_1(t)*x_2(t)$,则有：

$$y^{(1)}(t)=x_1^{(1)}(t)*x_2(t)=x_1(t)*x_2^{(1)}(t)$$

$$y^{(-1)}(t)=x_1^{(-1)}(t)*x_2(t)=x_1(t)*x_2^{(-1)}(t)$$

任意信号与冲激信号的卷积：任意信号$x(t)$与单位冲激信号$\delta(t)$的卷积仍为该信号本身，$x(t)$与$\delta(t-t_0)$的卷积相当于将信号$x(t)$延时t_0。

$$x(t)*\delta(t)=x(t)\qquad x(t)*\delta(t-t_0)=x(t-t_0)$$

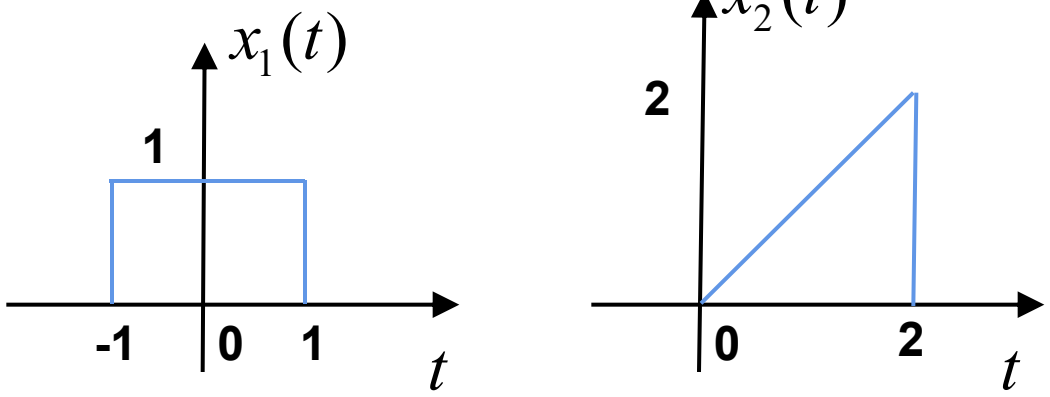
任意信号与阶跃信号的卷积：单位阶跃信号$u(t)$相当于积分器。

$$x(t)*u(t)=\int_{-\infty}^{\infty}x(\tau)u(t-\tau)d\tau=\int_{-\infty}^{t}x(\tau)d\tau$$

任意信号与冲激偶信号的卷积：冲激偶信号$\delta'(t)$相当于微分器。

$$x(t)*\delta'(t)=x'(t)$$

习题1.4 试求图所示的信号$x_1(t)$和 $x_2(t)$的卷积积分 $y(t)=x_1(t)*x_2(t)$



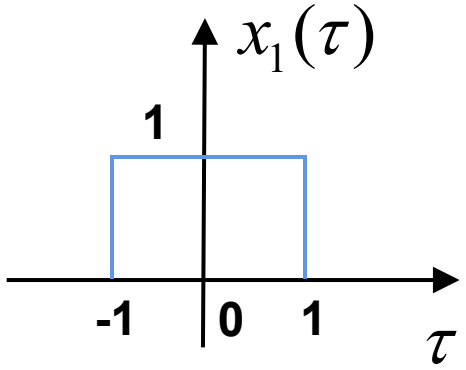
解：由图可得表达式：

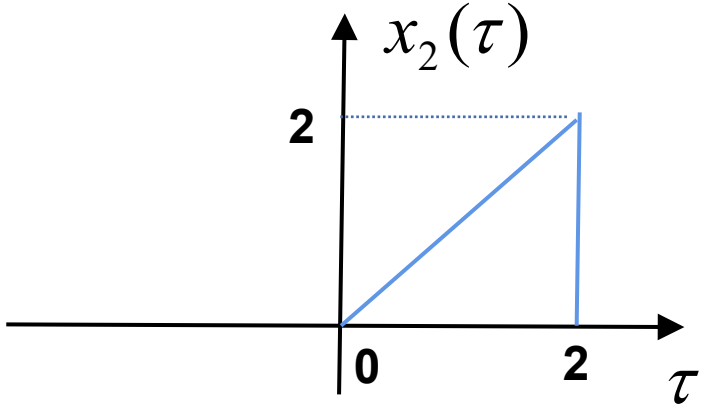
$$x_1(t)=\begin{cases}0 & t<-1,t>1\\ 1 & -1\le t\le 1\end{cases}$$

$$x_2(t)=\begin{cases}0 & t<0,t>2\\ t & 0\le t\le 2\end{cases}$$

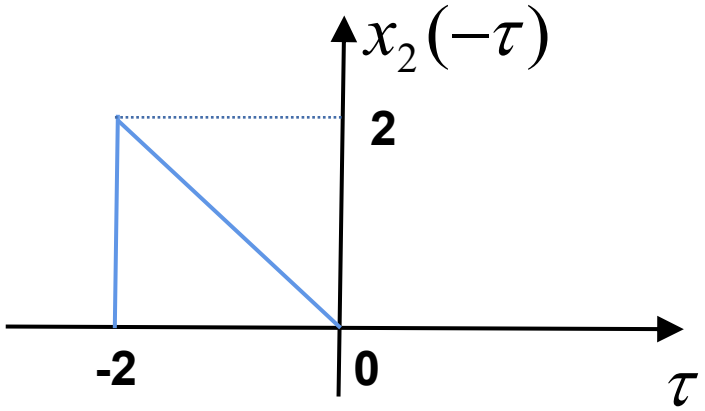
$$y(t)=x_1(t)*x_2(t)=\int_{-\infty}^{\infty}x_1(\tau)x_2(t-\tau)\,d\tau$$

自变量由t变为 τ

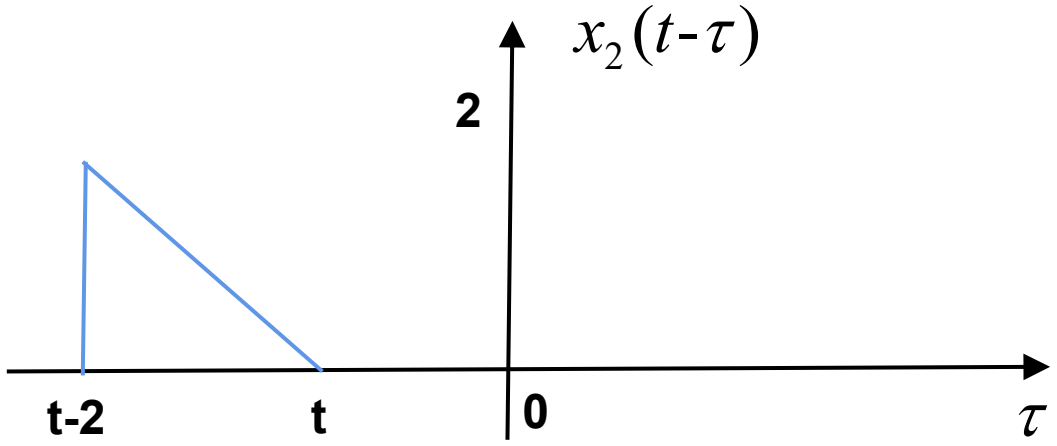




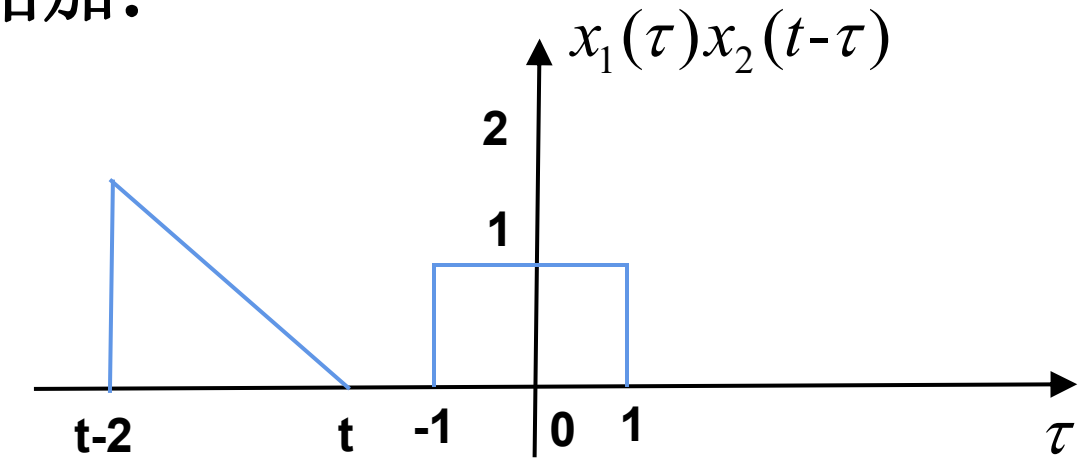
$x_2(\tau)$ 翻转为 $x_2(-\tau)$



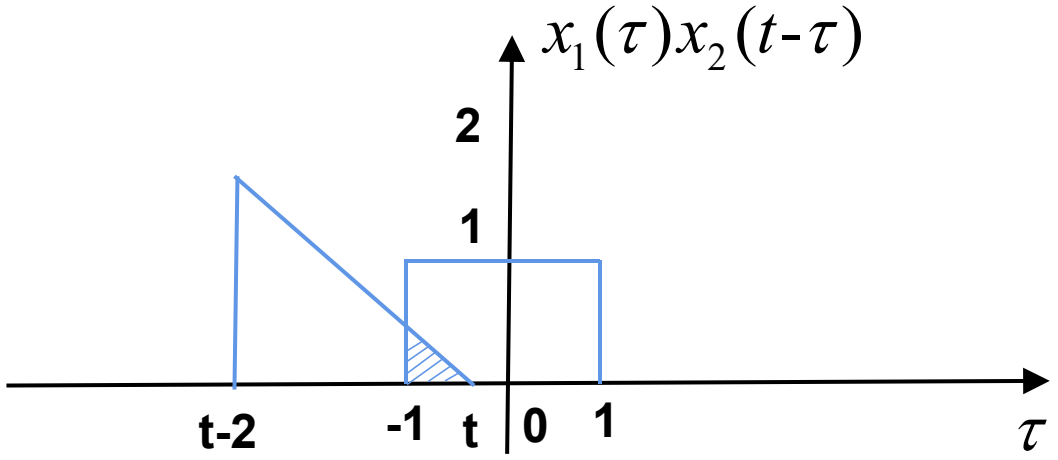
$x_2(-\tau)$ 移位t单位为 $x_2(t-\tau)$



相乘相加：



$$t \le -1, y(t) = x_1(t) * x_2(t) = 0$$

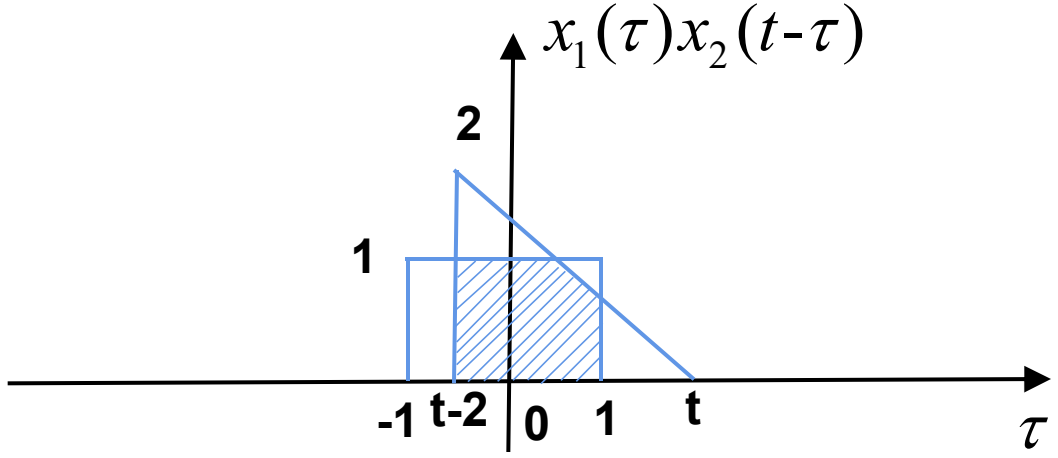


$$-1 \le t < 1,$$

$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau)\, d\tau$$

$$= \int_{-1}^{t} 1 \times (t-\tau)\, d\tau = -\frac{(t-\tau)^2}{2}\Bigg|_{-1}^{t} = 0 + \frac{(t+1)^2}{2}$$

$$= \frac{(t+1)^2}{2}$$

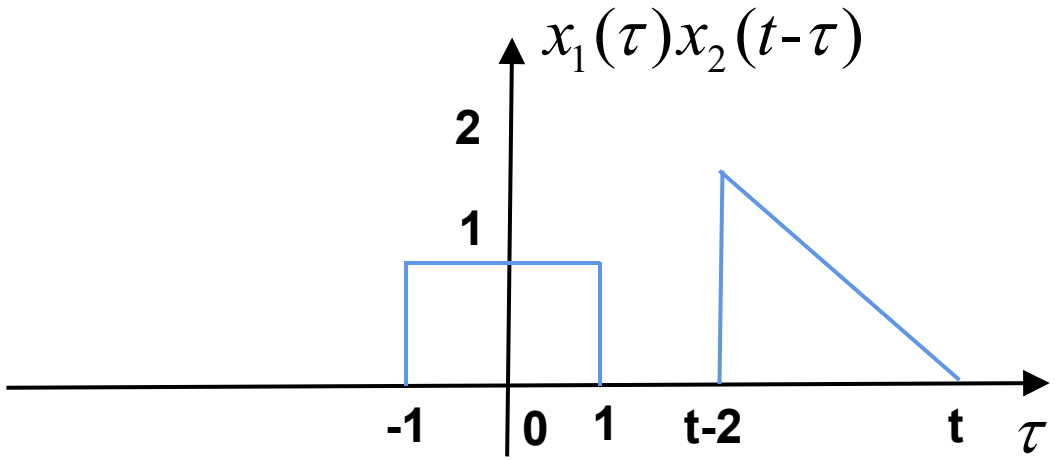


$1 \le t < 3,$

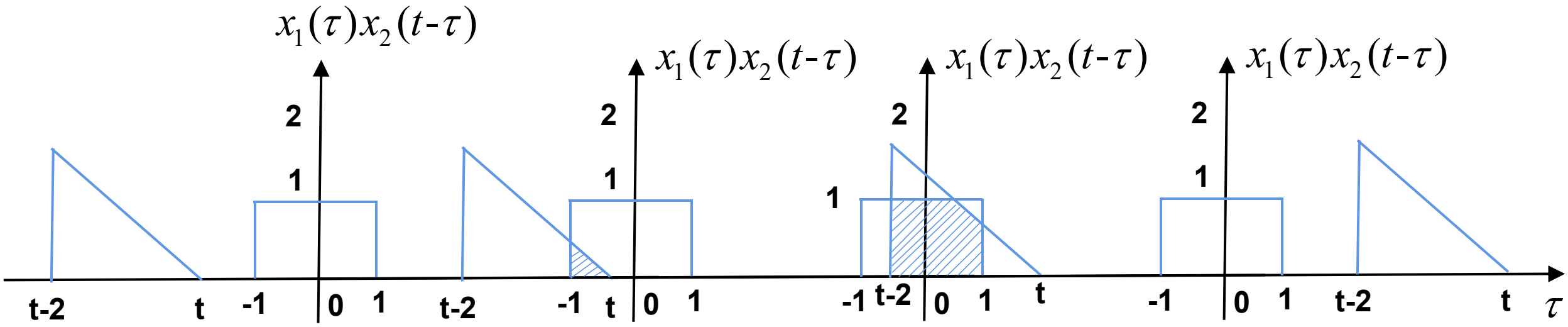
$$y(t) = x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau) x_2(t-\tau)\, d\tau$$

$$= \int_{t-2}^{1} 1 \times (t-\tau)\, d\tau = -\frac{(t-\tau)^2}{2}\Big|_{t-2}^{1} = -\frac{(t-1)^2}{2} + \frac{(t-t+2)^2}{2}$$

$$= -\frac{(t-1)^2}{2} + 2$$



$$t \ge 3, y(t) = x_1(t) * x_2(t) = 0$$



$$\therefore y(t) = x_1(t) * x_2(t) = \begin{cases} 0, & t < -1 \\ \frac{1}{2}(t+1)^2, & -1 \le t < 1 \\ -\frac{1}{2}(t-1)^2 + 2, & 1 \le t < 3 \\ 0, & t \ge 3 \end{cases}$$